La Cañada High School

Proposed Course Outline – LC Math 3

I. Course Title – LC Math 3

II. Grade Level(s) – Grades 9-12

III. Length/Credit – 1 Year - 10.0 units of Math Elective

IV. Preparations – Completion of LC Math 2, or their equivalents, with a grade of high C- or higher.

V. Course Description

Building on their work with linear, quadratic, and exponential functions, students extend their understanding of functions to include logarithmic, polynomial, rational, and radical functions in LC Math 3. This course includes standards from the conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Students work closely with the expressions that define the functions, competently manipulate algebraic expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

Some of the overarching ideas in the LCM3 course include: (1) relate arithmetic of rational expressions to arithmetic of rational numbers; (2) expand understandings of functions and graphing to include trigonometric functions; (3) synthesize and generalize functions and extend understanding of exponential functions to logarithmic functions; and (4) relate data display and summary statistics to probability and explore a variety of data collection methods. In addition to these central math concepts, students in LC Math 3 will also explore and model concepts verbally, numerically, algebraically, and graphically, construct viable arguments, engage in definitional thinking, continue to engage in inductive and deductive reasoning as it relates solving problems, making patterns, and applying math learning to real world problems, and the usefulness of Algebra for analyzing the world around us. The Standards for Mathematical Practice will play a prominent role in the course, with students continually developing their ability to reason and argue. Students will solve complex problems, and use tools such as graphing calculators and appropriate technology.

VI. Standards/ESLRs Addressed

1. Standards for Mathematical Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Number and Quantity

2. The Complex Number System Standards Abbreviation: N-CN

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.

Algebra

3. Seeing Structure in Expressions Standards Abbreviation: A-SSE

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

4. Arithmetic with Polynomials and Rational Expressions Standards Abbreviation: A-APR

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

5. Creating Equations Standards Abbreviation: A-CED

- Create equations that describe numbers or relationships.

6. Reasoning with Equations and Inequalities Standards Abbreviation: A - REI

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Represent and solve equations and inequalities graphically.

Functions

7. Interpreting Functions Standards Abbreviation: F- IF

- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

8. Building Functions Standards Abbreviation: F- BF

- Build a function that models a relationship between two quantities.

- Build new functions from existing functions.

9. Linear, Quadratic, and Exponential Models Standards Abbreviation: F-LE

- Construct and compare linear, quadratic, and exponential models and solve problems.

10. Trigonometric Functions Standards Abbreviation: F-TF

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Geometry

11. Expressing Geometric Properties with Equations Standards Abbreviation: G-GPE

- Translate between the geometric description and the equation for a conic section.

12. Circles Standards Abbreviation: G-C

- Find arc lengths and areas of sectors of circles.

Statistics and Probability

13. Interpreting Categorical and Quantitative Data Standards Abbreviation: S-ID

- Summarize, represent, and interpret data on a single count or measurement variable.

14. Making Inferences and Justifying Conclusions Standards Abbreviation: S-IC

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

15. Using Probability to Make Decisions Standards Abbreviation: S-MD

- Use probability to evaluation outcomes of decisions.

VII. Brief Course Outline

Essential Course Concepts: Quarter 1: Polynomial Identities, Equations, and Rational Expressions

- Know there is a complex number i such that i 2 = −1, and every complex number has the form a + bi with a and b real.
- Use the relation i 2 = −1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- Solve quadratic equations with real coefficients that have complex solutions.
- Interpret expressions that represent a quantity in terms of its context.
- Use the structure of an expression to identify ways to rewrite it.
- Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2) 2 = (x2 – y2) 2 + (2xy)2 can be used to generate Pythagorean triples.
- Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Common Core State Standards Addressed: N-CN 1, N-CN 2, N-CN 7, A-SSE 1, A-SSE 2, A-SSE 4, A-APR 1, A-APR 2, A-APR 3, A-APR 4, A-APR 6, A-CED 2, A-CED 3, A-CED 4.

Essential Course Concepts: Quarter 2: Rational and Inverse Functions and Related Equations

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context.
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- Write a function that describes a relationship between two quantities.
- Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2x3 or f(x) = (x + 1)/(x – 1) for x ≠ 1.

Common Core State Standards Addressed: A-REI 2, A-REI 3.1, F-IF 4, F-IF 5, F-IF 6, F-IF 7b, F-BF 1c, F-BF 4a

Essential Course Concepts: Quarter 3: Logarithmetic Functions and Trigonometric Identities

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

- Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. T-TF 4
- Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F-TF 5
- Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F-TF 6
- Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. F-TF 8
- Prove the half angle and double angle identities for sine and cosine and use them to solve problems. F-TF 10
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude
- Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Common Core Concepts Addressed: F-IF 7a, F-LE 4, F-TF 1, F-TF 2, F-TF 4, F-TF 5, F-TF 6, F-TF 8, F-TF 9, F-TF 10, F-IF 7e, F-BF 3

Essential Course Concepts: Quarter 4: Statistics and Probability

- Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?
- Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- Evaluate reports based on data.
- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
- Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

- Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
- Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
- Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Common Core Concepts Addressed: S-ID 4, S-IC 1, S-IC 2, S-IC 3, S-IC 4, S-IC 5, S-IC 6, S-CP 1, S-CP 2, S-CP 3, S-CP 4, S-CP 5, S-CP 6, S-CP 7

VIII. Methods of Assessment

Evaluation:

1. **Examinations**: Examinations are a critical component in monitoring comprehension and in preparing students in the development of key critical thinking, operational and computational skills, data analysis, and reading skills. The examinations in this course will follow the district examination policies. Exams will take the form of tests and quizzes given at appropriate instructional periods.
2. **Projects**: Students will be asked to complete both individual and group projects related to key concepts of this course.
3. **Homework**: Students will be assigned homework daily to provide independent practice opportunities in order to strengthen and deepen key concepts. **Homework intensity for this course expects that students will complete homework that will require approximately 60 minutes daily.**
4. **Class Participation**: Class participation will be graded on a weekly basis.
5. **Final Exam**: A final exam will be given at the conclusion of both first and second semester. It will be a comprehensive exam based upon the course of study completed during the year.

Grades:

All work will be assigned a point value, although not all work will receive a letter grade. Grades are based on total points accumulated during each grading period. I have structured the class in such a way to approximate your grade breaking down into the following percentages:

Examinations:	**75%**	**A= 90-100 %**
Homework/Classwork:	**10%**	**B = 79-89.9 %**
Final Exam:	**15%**	**C = 67- 78.9 %**
		D = 55-66.9 %
		F = Below 54.9 %

IX. Materials/Textbook(s)

Algebra 2, Student Common Core Edition, John Carter, et al. McGraw-Hill/Glencoe, 2016.

X. Seeking "a-f" Approval – Yes/No – Yes, this course will be submitted to the University of California for approval for the 2017-18 academic year in the subject domain "C" for mathematics.

XI. Seeking AP Class Approval – Yes/No – This course does NOT seek AP approval.