La Cañada High School

Proposed Course Outline – LC Math 4

I. Course Title – LC Math 4

II. **Grade Level(s) – Grades 10-12**

III. Length/Credit – 1 Year - 10.0 units of Math Elective

IV. Preparations – Completion of LC Math 3, or their equivalents, with a grade of high C or higher.

V. **Course Description**

LC Math 4 combines the topics of trigonometry, geometry, and algebra that are needed to prepare students for the study of calculus. Building on their understanding and work with logarithmic, polynomial, rational, and radical functions from LC Math 3, LC Math 4 will strengthen students' conceptual understanding of problems and mathematical reasoning in solving problems. The main topics in this Precalculus-based course are complex numbers, rational functions, polynomial functions, trigonometric functions and equations, trigonometric identities, inverse functions, vectors, matrices, sequences and series, parametric equations, and polar curves.

Some of the overarching ideas in the LC Math 4 course include: (1) represent complex numbers in the Cartesian plane, that operations with complex numbers have a geometric interpretation, and combine trigonometry and the geometry of the plane in order express complex numbers in polar form; (2) work with matrices and their operations; (3) find inverse matrices and use matrices to represent and solve linear systems; (4) work with trigonometric functions, investigating the reciprocal functions secant, cosecant, and cotangent, and the graphs and properties associated with trigonometric functions; (5) find inverse trigonometric functions by appropriately restricting the domains of the standard trigonometric functions and use them to solve problems that arise in modeling contexts; (6) define polar coordinates and curves and connect these to trigonometry and complex numbers; (7) graph rational functions and determine zeros, intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points; (8) analyze and graph polynomial functions; (9) work with vectors in the coordinate plane and in three dimensional space.

In addition to these central math concepts, students in LC Math 4 will also explore and model concepts verbally, numerically, algebraically, and graphically, construct viable arguments, engage in definitional thinking, continue to engage in inductive and deductive reasoning as it relates solving problems, making patterns, and applying math learning to real world problems, and the usefulness of advanced mathematics concepts for analyzing the world around us. The Standards for Mathematical Practice will play a prominent role in the course, with students continually developing their ability to reason and argue. Students will solve complex problems, and use tools such as graphing calculators and appropriate technology.

VI. Standards/ESLRs Addressed

1. Standards for Mathematical Practices

- Make sense of problems and persevere in solving them.

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Number and Quantity

2. The Complex Number System Standards Abbreviation: N-CN

- Perform arithmetic operations with complex numbers.
- Represent complex numbers and their operations on the complex plane.

3. Vector and Matrix Quantities Standards Abbreviation: N-VM

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Algebra

4. Seeing Structure in Expressions Standards Abbreviation: A-SSE

- Interpret the structure of expressions.

5. Arithmetic with Polynomials and Rational Expressions Standards Abbreviation: A-APR

- Rewrite rational expressions.

6. Creating Equations Standards Abbreviation: A-CED

- Create equations that describe numbers or relationships.

7. Reasoning with Equations and Inequalities Standards Abbreviation: A - REI

- Solve equations and inequalities in one variable.

Functions

7. Interpreting Functions Standards Abbreviation: F- IF

- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

8. Building Functions Standards Abbreviation: F- BF

- Build new functions from existing functions.

9. Trigonometric Functions Standards Abbreviation: F-TF

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Geometry

10. Similarity, Right Triangles, and Trigonometry Standard Abbreviation: G-SRT

- Apply trigometry to general triangles.

11. Expressing Geometric Properties with Equations Standards Abbreviation: G-GPE

- Translate between the geometric description and the equation for a conic section.

VII. Brief Course Outline

Essential Course Concepts: Quarter 1: Complex numbers, Quadratic Functions and Equations, nth roots and real exponents, Systems of Linear Equations and Inequalities, Functions from a Calculus Perspective; Power, Polynomial, and Rational Expressions; Exponential and Logarithmic Functions

- Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
- Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h gives the number of person-hours it taks to assemble n engines in a factory, the positive integers would be an appropriate domain for the function.
- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- Demonstrate an understanding of functions and equations defined parametrically and graph them.
- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(x+k)$, and for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- Find inverse functions.
- Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a non-zero rational expression; add, subtract, multiply, and divide rational expressions.
- Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

Common Core State Standards Addressed: N-CN.1, N-CN.2, N-CN.5, N-CN. 6.N-CN.7, N-CN.8, N-CN.9, F-IF.4, F-IF.6, F-IF.7, F-IF.10, F-BF.3, F-BF.4b, F-BF.4c, F-BF.4d, A-APR.2, A-APR.3, A-APR.6, APR.7, A-CED.1, A-CED.2, A-CED.3, A-CED.4.

Essential Course Concepts: Quarter 2: Trigonometric Functions; Trigonometric Identities and Equations; Systems of Equations and Matrices

- Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
- Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
- Prove the half angle and double angle identities for sine and cosine and use them to solve problems.
- Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- Prove the Laws of Sines and Cosines and use them to solve problems.
- Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and nonright triangles (e.g., surveying problems, resultant forces).
- Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- Add, subtract, and multiply matrices of appropriate dimensions.
- Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is non-zero if and only if the matrix has a multiplicative inverse.
- Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
- Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Common Core State Standards Addressed: F-TF.4, F-TF.6, F-TF.7, F-TF.9, F-TF.10, A-SSE.1, A-SSE.2, G-SRT.9, G-SRT.10, G-SRT.11, N-VM.6, N-VM.7, V-VM.8, V-VM.9, V-VM.10, V-VM.11, V.VM-12

Essential Course Concepts: Quarter 3: Systems of Equations and Matrices; Conic Sections and Parametric Equations; Vectors

- Represent a system of linear equations as a single matrix equation in a vector variable.
- Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 X 3 or greater).

- Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
- Given a quadratic equation, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola and graph the equation.
- Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.
- Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- Solve problems involving velocity and other quantities that can be represented by vectors.
- Add and subtract vectors.
- Multiply a vector by a scalar.

Common Core Concepts Addressed: A-REI.8, A-REI.9, G-GPE.3, G-GPE.3.1, V-VM.1, V-VM.2, V-VM.3, V-VM.4, V-VM.5

Essential Course Concepts: Quarter 4: Polar Coordinates and Complex Numbers; Sequences and Series; Limits; Derivatives; Integrals

- Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems.
- Students work with arthimetic sequences and series by investigating several different types of sequences.
- Students will use Sigma notion to represent and calculate sums of series.
- Students work with geometric sequences and series.
- Students apply binomial thereom to binomial expansion.
- Students demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity.
- Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability.
- Students know the defintion of the definite integral by using Riemann sums. They use this definition to approximate integrals.

Common Core Concepts Addressed: N-CN.2, N-CN.4, F-IF.11, Calculus 1, Calculus 4, A-APR.5, Calculus 13

VIII. Methods of Assessment

Evaluation:

1. **Examinations**: Examinations are a critical component in monitoring comprehension and in preparing students in the development of key critical thinking, operational and computational skills, data analysis, and reading skills. The examinations in this course will follow the district examination policies. Exams will take the form of tests and quizzes given at appropriate instructional periods.
2. **Projects**: Students will be asked to complete both individual and group projects related to key concepts of this course.
3. **Homework**: Students will be assigned homework daily to provide independent practice opportunities in order to strengthen and deepen key concepts. **Homework intensity for this course expects that students will complete homework that will require approximately 60 minutes daily.**

4. **Class Participation**: Class participation will be graded on a weekly basis.
5. **Final Exam**: A final exam will be given at the conclusion of both first and second semester. It will be a comprehensive exam based upon the course of study completed during the year.

Grades:
All work will be assigned a point value, although not all work will receive a letter grade. Grades are based on total points accumulated during each grading period. I have structured the class in such a way to approximate your grade breaking down into the following percentages:

Examinations:	**75%**	**A= 90-100 %**
Homework/Classwork:	**10%**	**B = 79-89.9 %**
Final Exam:	**15%**	**C = 67- 78.9 %**
		D = 55-66.9 %
		F = Below 54.9 %

IX. **Materials/Textbook(s)**

PreCalculus, Student Common Core Edition, John Carter, et al. McGraw-Hill/Glencoe, 2014. (LCUSD Approval Pending)

X. Seeking "a-f" Approval – Yes/No – Yes, this course will be submitted to the University of California for approval for the 2017-18 academic year in the subject domain "C" for mathematics.

XI. Seeking AP Class Approval – Yes/No – This course does NOT seek AP approval.