



Milpitas Unified School District Course Presentation

Title of Course: Integrated Math III Advanced

Course Duration: 1 Year

Credits: 10

Grade Level: 11

Department: Mathematics

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Prerequisites: : Completion of Integrated Math II Advanced with a grade of “B” or better and a teacher recommendation.

Sequence:

Magnet: No

Academy: No

UC/A-G Pending Approval

Course Catalog Information: Integrated Mathematics Math III Advanced is an integrated course intended for highly motivated third year math students. Students in this course will focus on unifying and applying the accumulation of learning that they have acquired from Integrated Math I and Integrated Math II. Integrated Mathematics Math III Advanced integrates high school conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability into a two semesters course.

Course Objectives

Integrated Math III Advanced covers all of the California Common Core standards for Integrated Math III, and is distinguished from the standard Integrated Math III in several ways:

- Optional advanced math standards for California Common Core Math III are covered, which are complex roots of polynomials, the Fundamental Theorem of Algebra, the Binomial Theorem, algebra manipulation of rational expressions, derivations of the law of sines, law of cosine, and triangle area formulas, and using probability to evaluate outcomes.
- The pace is accelerated to accommodate the additional topics.
- Students are expected to possess a high level of fluency in calculating with fractions and decimals, and a strong foundation in algebra, geometry, and statistics from prior coursework, as measured by a formal readiness assessment as well as the recommendation of the Integrated Math II Advanced teacher.
- Students will produce a formal mathematical research paper on the topic of their choice.
- The year-end final exam will be cumulative, integrating procedural and conceptual questions on the curriculum from entire year of the course.

The goal of Integrated Math III Advanced is to capitalize on the dedication of highly motivated and well-prepared students and empower them to develop a high level of procedural fluency, conceptual understanding, and critical thinking skills. Students will observe and apply at least one of eight Standards of Mathematical Practice in each lesson. Integrated Math III Advanced will prepare students for Honors Pre-Calculus or AP Calculus AB in the following school year.

Course Description

Integrated Math III Advanced is the third year course in the Integrated Common Core Pathway at Milpitas High School. Topics will include polynomial and rational functions, sequences and series, exponential and logarithmic functions, trigonometry and analyzing trigonometric functions, graphing, inverse, and reciprocal trigonometric functions, trigonometric identities, proving identities, probability & statistics, complex numbers, optimization and geometric modeling. In addition, the eight Mathematical Practice Standards will be implemented in the course as well. It is intended for students to be active learners and use critical thinking to solve problems and formulate certain properties and formulas on their own. All of these topics will be applied to mathematical modeling of real-world phenomena. Technology, particularly in the form of graphing calculators or online tools, will feature regularly in lessons and in student work.

Major Units of Study

Unit 1 Geometric Modeling

First, students will apply area, surface area, and volume formulas to solve problems involving density and masses. Second, students will be able to describe and draw cross sections the various cross sections of the solid. Finally, students will form solids of revolution in the coordinate plane.

Key Assignment: Students will use the knowledge of population density to measure how many people live within a given area. Locations of new schools are decided using geometric models based on population density. Students will be able to use these models help officials drawing new attendance boundaries when the school are built. The ultimate goals are to use area formulas to solve problems and use the formula for population density. Students make sense of mathematical models and use logical reasoning to create a report for the official detailing the new attendance boundaries for schools.

Unit 2 Linear and Quadratic Functions

First, students will learn to identify linear, quadratic, and absolute functions and describe combination of transformations of parent functions. Second, students will write functions representing combination of transformations and find lines of fit and lines of best fit for data. Third, students will describe and write transformations of quadratic functions, write equations of quadratic functions using characteristics of their graphs, and write quadratic equations to model data sets. Students will apply the knowledge to work with real-life applications.

Key Assignment: Students will collect temperature data for their city and will create a parabola to predict the temperature at midnight, after midnight, and next day. Students will be able to write a function that models the temperature over time and determine the time that is the coldest, hottest, or best temperature during the period? Students will be able to use technological tools strategically and make sense of mathematical models.

Unit 3 Polynomial Functions

First, students will learn to add, subtract, divide, and factor polynomials. Second, students will find the solutions of polynomial equations and zero of polynomial functions. Third, students will learn to use the Fundamental Theorem of Algebra. Finally, students will be able to graph polynomial functions, analyze graphs of polynomial functions, including transformations, and write polynomial function to model data.

Key Assignment: Students will be given maximum and minimum temperatures of different cities for each month except one month's temperatures will be missing. Students will be attempting to take the

remaining data points to fit a polynomial function and use their model to predict the maximum and minimum temperatures for the missing month. Students will use their polynomial model, math, and words to describe how they calculate the missing month's data and justify it using their model.

Unit 4 Rational Exponents and Radical Functions

First, students will learn to simplify and evaluate expressions using properties of rational exponents and properties of radicals. Second, students will graph and write transformations of radical functions. Third, students will solve equations containing radical and rational exponents and radical inequalities. Fourth, students will add, subtract, multiply, and divide radical and rational functions. Finally, students will find and verify inverses of nonlinear functions and graphs of inverse functions with the reflections in the line $y = x$.

Key Assignment: Students use function operations to write functions which model the following for a white rhino: the number of heartbeats per lifetime, the average amount of calories burned over its lifespan while resting, and the average amount of calories burned per heartbeat. Students will multiply and divide functions in their calculations. Students will complete the hand out questions by hand and use graphing calculator to check on their accuracy.

Unit 5 Exponential and Logarithmic Functions

First, students will write and graph exponential functions for base e and other bases. Second, students will define and evaluate logarithmic functions and use properties of logarithmic functions and exponential functions. Third, students will write and graph logarithmic functions and transformations of exponential and logarithmic functions. Fourth, students will use properties and formulas to evaluate, expand, or condense logarithmic expressions, and solve exponential logarithmic equations and inequalities. Finally, students will be able to apply exponential and logarithmic models for data sets.

Key assignment: Students will solve an exponential decay problem detailing the rate at which a drug is eliminated from the bloodstream of a patient. Students will need to take the decay rate of the drug and create a dosage plan that does not exceed a maximum amount of the drug in the bloodstream while maintaining a minimum level to ensure the effectiveness of a drug. Students will likely build a table and graph using different dosages and derive a logarithmic model/equation explaining the relationship between dosage and amount of the drug in the bloodstream.

Unit 6 Rational Functions

First, students will learn to classify direct and inverse variation equations and write inverse variation equations. Second, students will graph rational functions in three different forms. Third, students will learn to add, subtract, multiply, divide, and simplify rational expressions and

complex fractions involving rational expressions. Finally, students will solve rational equations and find inverses of rational functions.

Key assignment: Students will be able to apply rational functions to solve real-world problems. Students model a cost and profit function with rational functions. They investigate asymptotes graphically and analyze the end behaviors of the functions using the tables of values in their graphing calculators. They solve one function both graphically and algebraically. Then students analyze alternatives and draw conclusions about how to begin making a profit sooner.

Unit 7 Sequences and Series

First, students will learn to use sequence notation to write terms of sequences and write a rule for the n th term of a sequence. Second, students will use summation notation and find sums of finite arithmetic sequences, sums of finite and infinite geometric sequences, and partial sums of infinite geometric sequences. Finally, students will evaluate recursive rules for sequences and translate between recursive rules and explicit rules for arithmetic and geometric sequences.

Key assignment: Students write a recursive sequence to describe the growth of the walleye fish population in Lake Erie and categorize the growth according to levels established by marine biologists. They then write recursive sequences to describe possible interventions on the walleye population. Graphing calculators or spreadsheets are used to investigate growth rates and analyze the effectiveness of those interventions. Students use technological tools appropriately. They model real-life situations with mathematics.

Unit 8 Trigonometric Ratios and Functions

First, students will learn radian measure, and the six trigonometric functions that are defined in terms of a unit circle. Second, students will graph the sine and cosine trigonometric functions by plotting functional values for benchmark angles and the concept of periodic functions. Third, students learn the remaining four trigonometric functions that are deduced from knowing the relationship between these functions and sine and cosine. Finally, students will use trigonometric functions to model and solve real-life phenomena.

Key assignment: Students use right triangle trigonometry to analyze rope length and sailing height in a parasailing application. They use the concepts of similar triangles and ratio to define a rule of thumb to calculate the height of a parasail rider for a given length of tow line assuming the rider is being towed at a maximum safe speed.

Unit 9 Trigonometric Identities and Formulas

First, students will use and verify the fundamental trigonometric identities to simplify trigonometric expressions. Second, students will use the sum and difference formulas to evaluate and simplify trigonometric expressions and solve trigonometric equations. Third,

students will use the Law of Sines and Law of Cosines to solve triangles. Finally, students will learn to find area of triangles using the sine function or when all three sides lengths are known.

Key assignment: Students will explore sine functions in real-life scenarios. Students will create a sine function based on the fraction of the moon that is visible on a particular day of the lunar cycle. Students will also look at the amount of daylight based on the day of the year and the consumption of energy by U.S. households (high in winter for heat, low in spring, high in summer for air conditioning, and low in autumn). Students will produce a graph and derive a sine function describing the relationship and students will learn how many areas in life follow a sine pattern.

Unit 10 Data Analysis and Statistics

First, students will learn to calculate probabilities using normal distributions, and use z-scores and the standard normal table to find the probabilities. Second, students will distinguish between populations and sample, analyze hypotheses, analyze methods of collecting data, and recognize bias in sampling and in survey questions. Third, students will recognize how randomization applies to experiments and observational studies, and analyze experimental designs. Finally, students will estimate population parameters and find margins of error, and resample data using a simulation to analyze a hypothesis.

Key assignment: Students will collaborate on a project where they will create a research question, design their experiment or survey, plot and describe their data, and determine whether their results are statistically significant. Students will gain real-world skills into data collection, surveying, designing experiments, producing and describing results, and learning how research studies are conducted.

Standards Met

Number and Quantity

The Complex Number System

- Use complex numbers in polynomial identities and equations. [Polynomials with real coefficients; apply N.CN.9 to higher degree polynomials.]
 - (+) Extend polynomial identities to the complex numbers.
 - (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Algebra

Seeing Structure in Expressions

- Interpret the structure of expressions.

- Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
 - Extend polynomial identities to the complex numbers.
 - Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions

- Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.

Functions

Interpreting Functions

- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems.

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Graph all 6 basic trigonometric functions. CA

Geometry

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles.

- (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
 - (+) Prove the Laws of Sines and Cosines and use them to solve problems.
 - (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section.

Geometric Measurement and Dimension

- Visualize relationships between two-dimensional and three-dimensional objects.
- Modeling with Geometry
- Apply geometric concepts in modeling situations.

Statistics and Probability

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable.

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. (Include more complex situations.)

- (+) Use probability to evaluate outcomes of decisions. (e.g., drawing by lots, using a random number generator).
- (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Reading Materials

Title: Integrated Mathematics III; Publication Date: 2016; Publisher: Big Ideas Learning, LLC.

Authors: Ron Larson, Laurie Boswell.

Title: California Algebra 2; Publication Date: 2008; Publisher: Prentice Hall Authors: Bellman, Bragg, Charles, Hall Handlin, Kennedy.

Title: Algebra and Trigonometry-Structure and Method. *Book 2*; Publication Date: 2000;
Publisher: McDougal Littell Authors: Brown, Charles, Dolciani, Sorgenfrey, Kane.

Supplementary Instructional Materials:

1. Mathematics Vision Project Secondary Mathematics III

<https://www.mathematicsvisionproject.org/secondary-mathematics-i1.html>

2. Various on-line and teacher created resources