

## Santa Rosa City Schools Course Proposal: Math 3

Proposal Submitted By Dr. Rani Goyal, Director:

**Needs Statement:** Discuss how this course fits into your Site and/or the District's goals. Attach minutes of meetings where this course was approved at site or district leaderships meetings.

Math 3 is the third year of the integrated math program. As the school sites requested to move from the traditional math pathway to integrated, this is the next course in the pathway. .

**Graduation Requirements:** Specify which requirement is met. (High School only)

Mathematics

**UC a-g Requirements:** Specify which requirement is met. (High School only)

Mathematics "c"

Explain the rationale for course addition or modification. How does this fit in with district/site goals. Is this course replacing a current course, which course is it replacing and why? Will this course require new sections? Be explicit.

This course is replacing Algebra 2 across the district due to the decision by teachers to move from the traditional pathway to the integrated pathway.

**Explain the measurable learning outcomes**

Math 3 will focus on (1) extending knowledge of functions to include polynomial, exponential, logarithmic, radical and trigonometric functions and (2) synthesizing students' mathematical knowledge to create models and solve contextual problems.

The purpose of Mathematics 3 is to develop mathematically proficient students by building on the concepts students learned in Math 1 and 2 in the areas of Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability and extending their thinking in depth. Students will make connections between mathematical concepts and apply them to real world situations by creating, modeling and manipulating polynomial, exponential, logarithmic, radical and trigonometric functions. Students will generalize the geometric transformational effect to diverse functions.

Students will be expected to solve problems on their own and in groups to show they can apply what they have learned to non-routine situations. This course will prepare students for Trigonometry and Pre-Calculus and eventually Calculus and Statistics courses.

In Math 3, students:

- (1) Generalize the similarities between systems, such as polynomials and integers. The properties of integers provides the foundation for manipulating rational expressions as well as performing operations of polynomials.
- (2) Synthesize the transformation of function families, and apply this to polynomials and logarithms.
- (3) Recognize the role of bias, randomness and sample size in collecting data.
- (4) Use appropriate functions to model various situations to understand them better and make decisions

### Course Description (To be used in the course catalog)

Mathematics 3 is the third course of a three-year sequence of mathematics classes, which satisfy college entrance requirements and prepares students for Pre-Calculus, AP Calculus, and AP Statistics. Math 3 is a one year course which satisfies the Common Core Standards . It will strengthen and build on students' previous knowledge of Math 1 and Math 2.

### Detailed Course Design

(Course design should include the objectives, activities, assessments, and standards to be addressed in this course.)

#### **Daily Assignments:**

Daily assignments will be used as a way to assess student learning over time. Feedback will be provided by the teacher, but also through peer review, enabling students to communicate and express their mathematical understanding to a variety of audiences. Some of these daily assignments will include quick writes, quizzes, formative assessments, projects, technologically enhanced activities, and others that deepen understanding and strengthen communication skills.

#### **Assessments:**

Assessments will be given to measure how well students can integrate all knowledge learned in each unit. Assessments will have entry points and opportunities for challenge to meet the needs of all students at all levels. Summative assessments will contain longer constructed response questions to assess the student's math knowledge and communication skills. Benchmark tasks focusing on solving non-routine problems, communicating reasoning, and applications will be given throughout the year so that the students can demonstrate the ability to use content from various units to solve in depth problems, anchored in authentic contexts.

#### **Instructional Strategies:**

Students will experience a variety of instructional strategies within each unit. The students will use discovery and exploration to investigate new material and make conjectures and

connections. Students will engage in activities where they look for patterns, make claims, and validate their conjectures while critiquing and revising their work, and the work of their peers, to build understanding of mathematical concepts. Group and individual presentations will be used for the student(s) to report their findings to their peers and allow students to learn from each other. Direct instruction will be used when a topic or skill needs to be explicitly shown or modeled. Warm ups will be used to continuously integrate, spiral and review material, as will exit tickets to check for understanding. Students will have opportunities to work together and individually to accomplish tasks. Group norms and responsibilities will be developed in order to help facilitate high-quality discourse and provide opportunities for students to learn from each other. Students will be taught to use technology such as calculators, online tools, and other mathematical software to help derive solutions to equations, systems of equations and inequalities, linear regression, graphical representations, and functions as well as visualize algebraic transformations. Through the gradual release of responsibility from the teacher, students will take more responsibility and ownership of their learning and growth in order to understand how they can use the skills and knowledge they have learned. Tasks will routinely be used to increase the depth of thought and understanding for all students enrolled in Math 3.

## Unit 1: Functions

### Description:

In this unit, students will synthesize the geometrical transformations and the algebraic transformations of the parabola explored in prior courses to a wide variety of parent functions utilizing the graphing form of a function.

Students will deepen their understanding of solving and solutions as well as strengthen their techniques for solving equations, inequalities, and systems of equations and inequalities.

### Essential Questions:

- What does it mean to transform a graph or a function?
- What is a function family?
- How can you use a graph to solve an equation?
- What tools does a graphing calculator have to help you work with functions?

### Mathematical Goals:

#### Students will:

- Build on their previous understanding of the characteristics of a function, such as its domain and range.
- Build their proficiency with the graphing calculator to investigate functions.
- Develop a general method for transforming functions, including translations and dilations.
- Model situations with quadratic functions.
- Solve equations both algebraically and graphically.
- Solve problems by using systems of equations.
- Use systems of inequalities to solve problems of maximization.
- Model physical and geometrical relationships algebraically.

### Essential Content Standards:

- FI-F.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- FI-F.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- FI-F.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.7b,e: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-BF.1: Write a function that describes a relationship between two quantities.
- F-BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x)+k$ ,  $f(kx)$  and  $f(x+k)$  for specific values of (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context
- A-SSE.1: Interpret expressions that represent a quantity in terms of its context.
- A-SSE.2: Use the structure of an expression to identify ways to rewrite it.
- G-GPE.3.1: Given a quadratic equation of the form  $ax^2+bx+c=0$  use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle or parabola and graph the equation.
- A-REI.11: Explain why the  $x$ -coordinates of the points where the graphs of the equations intersect are the solutions of the equation; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

## Unit 2: Statistics

### Description:

Students build on probability concepts from earlier courses. They will use experiments, sampling and surveying to collect data which they then graphically display to answer critical questions. Students will then study variability, and perform hypothesis testing utilizing simulations. Students will be introduced to Normal Probability Distribution.

### Essential Questions:

- What are important considerations for sampling and surveying?
- How can data from a randomized experiment be used to compare two treatments?
- Can simulations be used to decide if differences between parameters are significant?

- What is the normal curve and how can you use it?

#### Mathematical Goals:

Students will:

- Learn the importance of randomness
- Collect data from surveys, observational studies, experiments and simulations
- Create and Interpret relative frequency histograms
- Use the normal probability density function to find areas under the normal curve and to determine percentiles.
- Use variability to determine margin of error
- Discern the general effect of sample size on variability
- Apply statistics to quality control and process control situations

#### Essential Content Standards:

- S-IC.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
  - S-IC.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
  - S-IC.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
  - S-IC.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
  - S-IC.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
  - S-IC.6: Evaluate reports based on data.
  - S-ID.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
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### Unit 3: Inverse Functions and Logarithms

#### Description:

Students will investigate multiple representations of inverse functions. Students will be introduced to logarithms as the inverse of exponentials. They will develop the properties of logarithms and use them to solve exponential application problems.

#### Essential Questions:

- How can you “undo” a function?
- Does every function have a corresponding inverse function?
- What is a logarithm, and how can they be used?
- How do you solve an exponential equation?

#### Mathematical Goals:

Students will:

- Graph a function given the graph of its inverse.
- Write rules for the inverse of linear, radical, rational, simple polynomial and exponential functions
- Prove and use simple laws of logarithms
- Translate between logarithms in any base using the definition of logarithm
- Simplify logarithmic numeric expressions and to identify their approximate values.
- Move between graphical and algebraic representations of polynomial functions.
- Apply their knowledge of transformations to graphs of logarithmic functions.
- Use logarithms to solve exponential equations.

Essential Content Standards:

- A-SSE.2: Use the structure of an expression to identify ways to rewrite it.
- F-BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x)+k$ ,  $f(kx)$  and  $f(x+k)$  for specific values of (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- F-BF.1: Write a function that describes a relationship between two quantities.
- F-BF.4: Find inverse functions.
- F-LE.4: For exponential models, express as a logarithm the solution to  $ab^c=d$ , where  $a, c$ , and  $d$  are numbers and the base is 2, 10, or  $e$ ; evaluate the logarithm using technology.
- F-LE.4.1(CA): Prove simple laws of logarithms.
- F-LE.4.2(CA) Use the definition of logarithms to translate between logarithms in any base.
- F-LE.4.3(CA): Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
- F-IF.7e: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

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## Unit 4: Polynomial Functions

Description:

Students will extend their knowledge of function families and algebraic transformations to polynomials. They will explore polynomials through multiple representations. Polynomial division will be developed to find zeros and factor polynomials. Exploring solutions to systems of polynomials will motivate the study of complex numbers.

Essential Questions:

- How can a polynomial be used to model some situations?
- What is a root or a solution of a polynomial? How can we locate them?

Mathematical Goals:

Students will:

- Understand that polynomials form an algebraic system analogous to the integers.
- Factor polynomials using polynomial long division.

- Apply the Remainder theorem to evaluate polynomial expressions.
- Write a polynomial function given particular roots.
- Move between graphical and algebraic representations of polynomial functions.
- Know the Fundamental Theorem of Algebra and use it when factoring polynomials.

#### Essential Content Standards:

- A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-APR.2: Know and apply the Remainder Theorem.
- A-APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-APR.4: Prove polynomial identities and use them to describe numerical relationships.
- A-APR.6: Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system.
- FI-F.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F-IF.7c: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- A-SSE.2: Use the structure of an expression to identify ways to rewrite it.

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## Unit 5: Trigonometric Functions, Analytic Trigonometry

### Description:

Students will be introduced to the unit circle, which they will use to extend the domain of the trigonometric functions to the real numbers. They will become familiar with radian measure. They will apply their function transformation skills to these new functions, and model periodic phenomena using sine and cosine functions.

Students will learn about inverse trigonometric functions and use them to solve trigonometric equations. They will learn about three additional trigonometric ratios (secant, cosecant and cotangent) and their corresponding functions.

### Essential Questions:

- What is the Unit Circle?
- Is a trigonometric function more than just a ratio?
- How far can you extend the domain of a trigonometric function?
- Can you transform trigonometric functions like you do with other functions?
- How can you model periodic phenomena?

- Do trigonometric functions have inverses?

### Mathematical Goals:

Students will:

- Expand their understanding of trigonometric functions, and move beyond the .
- Develop the Unit Circle by representing right triangles on the cartesian plane.
- Understand the concept of radian angle measure.
- Use the unit circle to understand the three basic trigonometric functions: sine, cosine, and tangent
- Explore the inverse sine, cosine, and tangent functions at a basic level.
- Define the reciprocal trigonometric functions and recognize their graphs.
- Derive the Pythagorean Identity.
- Explore the characteristics (amplitude, period, etc.) of these functions, and apply their knowledge of transformations of these function's graphs.
- Move between graphical and algebraic representations
- Use trigonometric functions to model periodic phenomena.

### Essential Content Standards:

- F-TF.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.2.1(CA): Graph all 6 basic trigonometric functions.
- F-TF.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- FI-F.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F-IF.7e: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-BF.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x)+k$ ,  $f(kx)$  and  $f(x+k)$  for specific values of (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

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## Unit 6: Series

Description:



Students will revisit and add to what they already know about arithmetic and geometric sequences. They will build on this knowledge and write both finite and infinite series and determine their sums.

#### Essential Questions:

- How can you easily find the sum of a sequence of numbers?
- How can an infinite series have a finite sum?

#### Mathematical Goals:

Students will:

- Employ summation notation for series.
- Develop and use formulas for the sum of finite arithmetic and geometric series.
- Develop and use a formula for the sum of an infinite geometric series,

#### Essential Content Standards:

- A-SSE.2: Use the structure of an expression to identify ways to rewrite it.
  - A-SSE.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*
  - A-APR.4: Prove polynomial identities and use them to describe numerical relationships.
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## Unit 7: Algebra in Three Variables

#### Description:

In this unit, students will expand their understanding of graphing equations and systems of equations to three variables and will broaden their understanding of solutions to include solutions to systems in three variables.

#### Essential Questions:

- How do you solve a system of equations in three variables?
- How can you use three-variable systems to solve problems?

#### Mathematical Goals:

Students will:

- Graph equations in three dimensions.
- Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- Apply geometric methods to solve design problems
- Solve systems of equations in three variables by hand and by using appropriate technology
- Fit quadratic curves using three-variable systems of equations.

### Essential Content Standards:

- A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.7e: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- G-MD.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- G-MG.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- G-MG.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

### Key Assignments:

Students will engage with tasks involving a range of mathematical content standards and practices from the standards, such as the tasks cited below.

#### Mathematical Practices Addressed:

MP1: Make sense of problems and persevere in solving them

MP2: Reason abstractly and quantitatively

MP3: Construct viable arguments and critique the reasoning of others

MP4: Model with mathematics

MP5: Use appropriate tools strategically

MP6: Attend to precision

MP7: Look for and make use of structure

MP8: Look for and express regularity in repeated reasoning

#### Unit 1: Functions

Task 1: Graphing Stories (Buzzmath)

Task 2: Waterline (Desmos)

#### Unit 2: Statistics

Task 1: Interpreting Data: Muddying the Waters (Shell Center)

Task 2: Types of Statistical Studies (Illustrative Mathematics)

Task 3: Tennis Balls in a Can (Illustrative Mathematics)

#### Unit 3: Inverse Functions and Logarithms

Task 1: Carbon 14 Dating (Illustrative Mathematics)

Task 2: Doctor Dedman (CPM)

#### Unit 4: Polynomial Functions and Rational Expressions

Task 1: Generating Polynomials from Patterns (Shell Center)

Task 2: Representing Polynomials Graphically (Shell Center)

Unit 5: Trigonometric Functions

Task 1: Trigonometric Ratios and the Pythagorean Theorem (Illustrative Mathematics)

Task 2: Ferris Wheel (Shell Center)

Unit 6: Series, Binomial Theorem

Task 1: Domino Skyscraper (Dan Meyer Three Acts)

Task 2: YouTube Explosion (Illustrative Mathematics)

(+)Task 3: Powers of Eleven (Illustrative Mathematics)

Unit 7: Algebra in Three Variables

Task 1: Global Positioning Systems I (Illustrative Mathematics)

Task 2: Representing 3D objects in 2D (Shell Center)

Budget- budget figures must be included even if they are an estimate.

Projected Costs	Start-up	Ongoing
Personnel (Not to include classroom instructor unless a new section is needed)	NA	
Instructional Material Supplies per student (textbooks, software, etc.)	NA	
Services (training, equipment maintenance, contracts, etc.)	NA	
Capital Outlay (remodeling, technology, etc.)	NA	
<b>Total Projected Costs</b>		

Instructional Materials- must include estimate for new materials even if none have been selected. Place in chart above.

Type	Publisher	Title	ISBN	Author	Copyright	# Have/Need

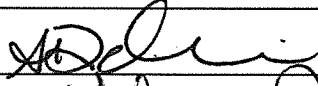
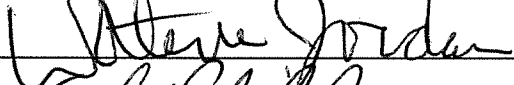
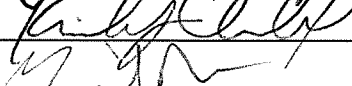

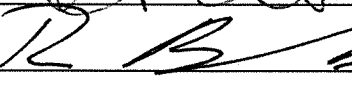

Funding Source(s) for Costs and Instructional Materials

Grants (indicate specific grant and grant timeline)	
Categorical Funds (include related programs)	
Career Technical Education (must be for an approved CTE course)	
Department Funds	
Other (be specific)	

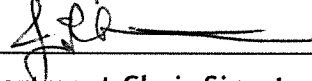

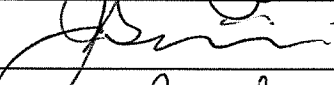
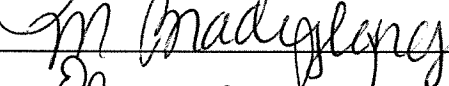

#### Appendix of Additional Documents

<i>* Required additional documents include meeting minutes where the course was discussed and approved</i>

#### District Principal Review and Approvals:

Principal's Signatures	Site	Approved / Not Approved
	PHS	approved
	RHS	approved
	SRHS	Approved
	EAHS	Approved
	MCHS	Approved
	MHS	Approved

#### District Department Chair Review and Approvals:

	RHS	Approved
Department Chair Signatures	Site	Approved / Not Approved
	SRHS	Approved
	EAHS	Approved
	MCHS	Approved
	MHS	Approved